Int. J. Heat Mass Transfer. Vol. 5, pp. 109–112. Pergamon Press 1962. Printed in Great Britain.

PIONEER PAPERS IN CONVECTIVE MASS TRANSFER

3. W. K. LEWIS: The evaporation of a liquid into a gas, *Mechanical Engineering*, **44**, 445 (1922). Reprinted by permission of The American Society of Mechanical Engineers.

Editor's Foreword

Nowadays the term "Lewis Number" is applied to the ratio of the diffusion coefficient of a component to the thermal diffusivity of the mixture. In an earlier usage however the term signified the ratio of a mass transfer coefficient times a specific heat to a heat transfer coefficient. In either case, the dimensionless quantity in question measures the relative ease of mass and of heat transfer in the same situation; its name is a tribute to the author of the paper reprinted below. In this paper, W. K. Lewis "proves" the above relation between the coefficients (i.e. $ks/h = 1$) and adduces experimental data in support. We now know that the proof contains a fallacy and that the relation between the coefficients holds only when the Lewis Number (in the modern sense) is equal to unity. Fortunately, this condition is closely fulfilled in most mass-transfer processes involving gases; Lewis's result is therefore widely and justifiably used in the design of cooling towers, in the analysis of fuel combustion and in many other problems.

D.B.S.

THE EVAPORATION OF A LIQUID INTO A GAS

W. K. LEWIS*

The author investigates the mechanism of the evaporation of a liquid into a gas as applied to such processes as are found in gas scrubbers, humidifiers, dehumidifiers, water coolers, air driers, etc. He establishes a formula for calculating the humidity of air from wet- and dry-bulb thermometer readings, and shows that the coefficient of heat transfer divided by the coefficient of diffusion equals the humid heat of the gas.

A LARGE amount of work has been done on the evaporation of water into air at temperatures below the boiling point. The dynamic equilibrium corresponding to the evaporation of water into air counterbalanced by the flow of heat from the air into the water is the basis of wet-bulb thermometry, the most useful method of determining the humidity of air.[1] In 1886 Desmond Fitzgerald[2] pointed out that the rate of evaporation of water into air is a function of the difference in partial pressure between the moisture in equilibrium with the evaporating water and the actual moisture content of the air in contact with it. It is true that Fitzgerald did not assume the rate of evaporation linearly proportional to this difference, but added a small correction term proportional to the square of the difference. Barrows and Babb[3] made a large number of determinations of evaporation from the surface of Maine lakes, and while their experimental determinations were subject to a large percentage variation, none the less their results substantiate this proportionality. More recently Willis H. Carrier[4] has shown that the rate of water evaporation is, within experimental error, proportional to partial-pressure difference. The following is an analysis of the mechanism of such evaporation.

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1. Leslie Nicholson's Journal, vol. 3, p. 401. August, Pogg. Ann., vol. 5, p. 69, 1825; Apjohn, Trans. Royal Irish Acad., vol. 17, p. 275, 1834; Weilermann, Meteorol. Zeit., vol. 12, pp. 268 and 368, 1877; Maxwell, Zeit. f. Meteorol., vol. 16, p. 177, 1881. D. Chowolson, Traité de Physique, vol. 3, part 3, p. 807, 1911.
2. Journal Am.Soc.C.E., 1886.
3. U.S. Dept. of Interior, Washington, D.C., Water Suppl., Bulletin No. 279.
4. Am.Soc. Refrigerating Engineers' Journal, 1916, vol. 2, no. 6, p. 25.

Paper presented at the Spring Meeting, Atlanta, Ga., May 8 to 11, 1922, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Slightly abridged. All papers are subject to revision.

PRELIMINARY ASSUMPTIONS

For purposes of formula derivation assume a long tunnel through which unsaturated air is flowing at constant mass velocity. The walls of this tunnel are non-conductors of heat. Along the bottom of the tunnel is placed a mat or wick permanently wet with water supplied from below as evaporation takes place above. The water is furnished to this wick at every point at a temperature exactly equal to that of the water on the upper surface of the wick at that point. There is no motion of the liquid water parallel to the axis of the tunnel. The mass velocity of the air over the water is constant and sufficiently low so that heat generated by friction may be neglected.

The unsaturated air entering this tunnel will become humidified in passing through it owing to the evaporation of water. In consequence, the temperature of the air will fall, and if the tunnel be sufficiently long, the water and air will ultimately come to equilibrium.

NOTATION

In the formula to be derived, the following notation is used:

A = Area of liquid in contact with gas;
H = Absolute humidity of gas, or parts by weight of vapor per part by weight of vapor free gas;
h = Surface coefficient of conductivity of heat between gas and liquid or B.t.u. per unit time per unit surface area per unit temperature difference;
k' = Coefficient of diffusion, or parts by weight of vapor diffused per unit time per unit area per unit absolute humidity difference;
k = Coefficient of diffusion, or parts by weight of vapor diffused per unit time per unit area per unit vapor pressure difference;
p = Partial pressure of vapor;
P = Total pressure of vapor and vapor-free gas (i.e., barometer);
θ = Time;
t = Temperature;
r = Latent heat of vaporization;
s = Humid heat, or number of heat units necessary to change the temperature of unit weight of vapor-free gas, *plus the vapor it contains*, one degree;
W = Weight of liquid evaporated.

INTERACTION OF WATER WITH AIR

Now consider for the moment the conditions at any given point along the length of this tunnel. At this point the temperature, absolute humidity, and pressure of water vapor in the air will be represented by t, H and p. Since the apparatus is continuous in its operation these conditions will remain unchanged at this particular point, but will vary from point to point along the tunnel. The corresponding quantities representing the condition of the liquid water in contact with the air at this particular point are t_w, H_w and p_w.

The mechanism of interaction of the water with the air is as follows: There exists over the water what is equivalent to a stationary film of air, which insulates the water from the main body of the air. Through this air film heat is diffusing from the air into the water and through the same film there is diffusing, in the opposite direction, the water vapor formed by evaporation on the surface of the liquid. This evaporation cools the surface of water, and, since it is available from no other source, heat must be supplied solely by diffusion from the air. The heat of vaporization must therefore be quantitatively compensated by the heat flow through the surface film, and the rate of evaporation is limited by the rate of diffusion of vapor through the same film.

From the foregoing one can immediately write the following equations:

$$-\frac{\mathrm{d}W}{A\mathrm{d}\theta} = k'(p_w - p) \qquad (1)$$

$$\frac{\mathrm{d}Q}{A\mathrm{d}\theta} = h(t - t_w) \qquad (2)$$

$$\mathrm{d}Q = -r_w\mathrm{d}W \qquad (3)$$

Whence

$$p_w - p = \frac{h}{k'r_w}(t - t_w) \qquad (4)$$

This last equation is the one normally used for calculating the humidity of air from wet- and dry-bulb thermometer readings. In it, variation in r_w is neglected and the term $h/k'r_w$ is assumed constant. For p in millimeters of mercury and t in deg. cent., it equals 0·5. The equation implicitly assumes that the cooling of the air is differential, i.e., so small in the neighborhood of the point in question that the actual changes in temperature and humidity of the air, t and p (or H), are negligible.

Since

$$p = P\frac{\frac{H}{18}}{\frac{H}{18}+\frac{1}{29}} \qquad (5)$$

$$t - t_w = \frac{k'r_w}{h}P\left(\frac{\frac{H_w}{18}}{\frac{H_w}{18}+\frac{1}{29}} - \frac{\frac{H}{18}}{\frac{H}{18}+\frac{1}{29}}\right) \qquad (6)$$

Where H is small, as is usually the case below 150 deg. fahr., $\frac{H_w}{18}$ and $\frac{H}{18}$ are negligible compared with $\frac{1}{29}$, and one may write, as a close approximation,

$$t - t_w = \frac{kr_w}{h}(H_w - H) \qquad (7)$$

where

$$k = 29k'\frac{P}{18}$$

It is obvious that h and k depend on the thickness of the air film and are therefore functions of the velocity of the air. It is, however, equally obvious that if air velocity be increased sufficiently to double the one, the other will double also. The ratio of h/k therefore remains constant, independent of velocity. This explains why the reading of a wet-bulb thermometer is uninfluenced by the velocity of air passing it, provided the velocity is sufficient so that any heat lost by radiation is negligible in comparison with that picked up by conduction.

THE RELATION $h/k = s$

Now consider the change in humidity and temperature of the air as it moves along the tunnel. Starting at the same point previously considered, the air will drop in temperature by an amount dt and increase in humidity by an amount dH. The heat given up by cooling must correspond to the heat of vaporization of the water picked up, i.e.,

$$-sdt = r_w dH \qquad (8)$$

whence, assuming constancy of s and r_w,

$$H = -\frac{s}{r_w}t + \text{const.} \qquad (9)$$

Assuming the tunnel indefinitely long, the air will ultimately become saturated at some temperature t_e, and humidity H_e. Since these conditions represent equilibrium between the air and the water, evaporation will cease, and t_e and H_e are therefore the constant, fixed end-points of the process. Inserting these limits,

$$H_e - H = \frac{s}{r_w}(t - t_e).$$

We have now derived two formulas connecting H and t, both applying to this same process of evaporation, i.e.,

$$H_e - H = \frac{s}{r_w}(t - t_e)$$

and

$$H_w - H = \frac{h}{kr_w}(t - t_w)$$

These two expressions must therefore be identical. By the method of undetermined coefficients this can be true only provided the corresponding coefficients are equal, i.e.,

$$\frac{s}{r_w} = \frac{h}{kr_w}, \text{ or } s = \frac{h}{k}$$

and $t_w = t_e$, a constant: and $H_w = H_e$, also constant.

So far the discussion has been limited to water and air. Obviously, however, the same relationships must apply to any liquid and any gas with which its vapor is mixed.

The first of these equations, $h/k = s$, states that the coefficient of heat transfer divided by the coefficient of vapor diffusion through the gas film is constant, and equal to the humid heat of the gas. By means of formula (4) the ratio h/k can be calculated from the observed wet- and dry-bulb temperatures for any vapor–gas mixture of a known gas humidity. The experimental determinations of wet-bulb temperatures for water–air, water–carbon dioxide, toluol–air and chlorbenzol–air, and calculations for h/k given in Table 1 show that this ratio is in all cases substantially equal to the humid heat of the entering gas, which in this case was identical with the specific heat because the gas which was used was vapor free.

Table 1. *Values of h/k for various vapor–gas mixtures*

	h/k calculated from experimental results	Specific heat of gas
Water–air	0·236	0·238
Water–carbon dioxide	0·217	0·220
Toluol–air	0·238	0·238
Chlorbenzol–air	0·248	0·238

We have therefore demonstrated that, granting substantial constancy of s and r_w, and assuming H to be small, the ratio of the coefficient of diffusion of heat to that of any vapor through the gas film on the surface of the liquid is equal to the "humid" heat of the gas. Furthermore, during "adiabatic"[1] evaporation of a liquid into a gas, the liquid being in dynamic equilibrium with the gas the temperature of the liquid remains unchanged throughout the process and the end-point of the process is reached when the gas has cooled itself to saturation at a temperature identical with that of an ordinary wet-bulb thermometer.[2]

IMPORTANCE OF THE RELATIONSHIP $h/k = s$

The importance of the relationship $h/k = s$ is very great. The term s, the humid heat, may be readily calculated for any case regardless of whether the problem is primarily one of heat transfer or of diffusion. Hence if the heat-transfer coefficient h has been experimentally determined for a certain type of apparatus operating under definite conditions, the coefficient of diffusion equals h/s, and the capacity of this same apparatus may be predicted when functioning in diffusion processes, e.g. as a gas scrubber. Conversely, if k and s are known for definite conditions, h equals ks; in other words, one can predict the performance of a given apparatus for heat transfer from data upon the same equipment functioning as a scrubber.

These processes of diffusion of heat and of vapor are at the basis of the performance of all such equipment as humidifiers, dehumidifiers, water coolers, gas scrubbers, air driers, light oil stripping columns, and the like. The above relationships make it possible to study the performance of such equipment on a more rational basis than hitherto and to compare the effectiveness of different types of equipment even when the data on the individual types are obtained under widely varying conditions. The Department of Chemical Engineering, Massachusetts Institute of Technology, expects to publish in the near future a series of articles showing various applications of these relations.*

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[1] W. H. Carrier, Journal Am.Soc.M.E., 1912, p. 1321.

[2] *Ibid.*

* *Editor's footnote:* It was in one these, namely *Mech. Engng* vol. 55 (1933), p. 567, that Lewis himself pointed out the fallacy in the argument of his 1922 paper. Before turning to the later paper for confirmation, readers might like to make up their own minds about where the fallacy lies.

ERRATA

H. L. Evans: Mass transfer through laminar boundary layers—6. Methods of evaluating the wall gradient (b_0'/B) for similar solutions; some new values for zero main-stream pressure gradient, *Int. J. Heat Mass Transfer*, **3**, No. 4, 321–339 (1961).

The following corrections should be made to formulae—

(a) Equation (25) should read:

$$e = \frac{\beta}{f_0^4} \tag{25}$$

(b) The penultimate term of equation (33) should read:

$$-5775e^3$$

(c) Equation (54) should read:

$$\frac{v_0\Delta_2}{K} = \frac{1}{(1/\sigma f_0^2) - (1 + \sigma)} \tag{54}$$